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OPTIMAL 16-RUNS FOLDOVER DESIGNS

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Abstract:

In this paper, various foldover designs are constructed from 16-run two-level fractional factorial designs developed by Li and Lin (2003). They state that for any core foldover plan there exist 2^{n-p}equivalent foldover plans. These equivalent foldover plans yield designs that are identical in terms of minimum aberration optimality criterion. Minimum aberration criterion alone does not necessarily guarantee comparable performance under practical considerations. Therefore, these equivalent plans obtained from 16 run foldover designs are further compared based on additional criteria of clear factor interaction, trend resistance and number of level changes. The objective of the research is to find optimal foldover designs using four criteria namely minimum aberration, clear factor interactions, minimum number of level changes and trend resistance.

Subject Classification Code: 62K05

Keywords: Fractional factorial design, Foldover plans, Equivalent designs, Number of level changes, trend free designs.

1. Introduction

In a factorial design, responses are observed for every combination of factor levels. There is an exponential increase in the runs for a complete replicate of the design, with the increase in the number of factors. Sometimes conducting a full factorial experiment is costly and time-consuming. Therefore, in such situations, it is recommended to use fractional factorial design (FFD) which is a fraction of a complete factorial experiment. A good choice of a fraction of a full factorial design allows the experimenter to conduct the experiment with fewer runs and

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simultaneously allows the estimation of large number of effects. It is experimenter's requirements and the availability of resources that helps to decide which fraction should be selected from the full factorial experiment. Thus, selecting a design with appropriate properties is an important criterion for experimentation.

Box and Hunter (1961) were the first ones to address the problem of design selection. They introduced the concept of resolution as a goodness criterion for design. However, the resolution criterion alone is insufficient to identify the best design because the designs which may have same resolution may not be equally good, in terms of estimation of different numbers of main effects (*m.e's*) and clear two-factor interactions (2 *fi's*). Fries and Hunter (1980) suggested minimum aberration (*MA*) criterion, to distinguish between the designs having the same resolution. Wu and Chen (1992) proposed the concept of clear *m.e.'s* and clear 2 *fi's*. They proposed that the design with a greater number of clear *m.e.'s* and 2 *fi's* may be superior to the design with *MA*. Chen et al. (1993) provided a catalogue of designs to make it easier to find the best design based on several criteria (resolution, *MA* and clear 2 *fi's*).

All 2^{n-p} FFDs with resolution III or IV may not estimate an equal number of clear 2 *fi's* for a given number of runs. Chen and Hedayat (1998) gave a unified geometrical study for regular 2^{n-p} designs, to determine when resolution III or IV designs can have clear 2 *fi's*. An efficient technique for constructing 2^{n-p} designs with greater clear 2*fi's* was introduced by Tang et al. (2002) , and deduced both upper and lower bounds on the maximum number of clear 2 *fi's* in 2^{n-p} FFD of resolution III and IV.

Improved foldover fractions for 2^{n-p} resolution III designs were given by Li and Mee (2002). These foldover fractions increase the resolution of the design to IV and separate some of the aliased 2 *fi's* to permit estimation of many more 2 *fi's*. Li and Lin (2003) discussed the optimal foldover plan for two-level FFD by changing the signs of one or more columns of the initial design and defining a foldover plan to select the optimal design based on *MA* criterion.

While examining the run orders an experimenter may encounter two types of problems, first how to select a trend-resistant run order and second change of factor levels from one run to next. Therefore, it is ideal to have a run order that is trend-free while also having the fewest number of level changes (NLC).

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The idea of systematic order in an experimental design was first introduced by Cox (1951). Draper and Stoneman (1968) were the first to use number of factor level changes and time trend as criteria to determine the run order of two–level factorial designs. Daniel and Wilcoxon (1966) proposed an important method for determining the run orders of certain 2^n factorial designs where the main effects are orthogonal to linear and quadratic time trends. Cheng and Jacroux (1988) constructed run orders of 2^n complete and 2^{n-p} FFDs where the estimates of main effects and *2fi'sare* orthogonal to some polynomial trends. Joiner and Campbell (1976) constructed designs in which each factor changed the level from one run to the following run with a predetermined probability. Cheng (1985) investigated the order of the run with trend resistance and minimum cost. Coster and Cheng (1988) considered a generalized foldover scheme to construct trend-resistant FFDs. Dickinson (1974) utilized a computer program to examine the designs which required fewest NLC among the factors and choose those in which the correlations between linear time trend and the main effects were small. Cheng and Steinberg (1991) gave a reverse foldover algorithm, which is a modification of an algorithm given by Cheng (1985) and Coster and Cheng (1988), for constructing the run order of two–level FFD that maximizes the NLC and also indicate how to modify the algorithm to get the linear trend free (LTF) designs with the maximum NLC.

Cheng et al. (1998) applied the reverse foldover algorithm to construct run orders of 2-level factorial designs that achieved extreme (maximum and minimum) numbers of level changes. Later, Adekeye and Kunert (2006) this line of work by comparing the run orders of un-replicated 2^{n-p} designs in the presence of a time trend.

Singh et al. (2013) used linear codes to generate linear trend-free run orders for orthogonal arrays. Singh et al. (2014 a) constructed trend-resistant orthogonal arrays with the parity check matrix of a linear code. Singh et al. (2014 b) presented a simple methodology to construct the FFDs with s, s≥ 2 levels, having some effects (main andtwo-factor interactions) linear trend-free using finite field. This method was used to construct the blocked FFDs and mixed FFDs with some linear trend free effects. Singh et al. (2016 a, 2016 b) presented a method of constructing LTF FFDs and blocked FFDs with linear codes.

Bhowmik et al. (2019) presented the construction of response surface designs with minimum level changes in the run sequences. Bhowmik et al. (2020) presented construction of costeffective factorial designs with the minimum number of changes in the factor levels. Thapliyal and Budhraja (2020) proposed trend-resistant run order for 2-level factorial designs based on a symmetric balanced incomplete block design (BIBD).

Hilow and Maayteh (2021) used normalized Hadamard matrices of size $2^n \times 2^n$ for the construction of three types of run orders with minimum cost for full 2^n factorial experiments. These run sequences require level changes that are minimum in number in the factors and factor effects that are orthogonal to the linear/quadratic time trend.

Building upon the 16-run two level FFDs developed by Li and Lin (2003) this paper investigates the construction evaluation of foldover designs derived from these plans. They established that each core foldover plan has 2^{n-p} equivalent foldover plans which are identical with respect to *MA* criteria. However, equivalence under MA alone does not ensure comparable performance in practical experimental setting.

Motivated by the limitation of the equivalent designs proposed by Li and Lin (2003) the present study systematically examines these equivalent foldover plans using criteria (i) clear *m.e.'s*, and *2fi's,* (ii) trend resistance and (iii) number of level changes. In this paper the authors choose optimal designs depending upon different requirements. The primary objective of the paper is to identify optimal foldover plan using the four criteria mentioned.

The paper is organized as follows. Section 2, contains the preliminaries. Section 3, evaluates the designs using the criteria of MA and clear *m.e.'s* and clear 2 *fi's* and determines the designs optimal with respect to both the criteria. Section 4, discusses the construction of equivalent foldover designs which are then compared based on minimum number of level changes and LTF factors.

2. Preliminaries

A $2^{\mathbf{n}-p}$ fractional factorial design is said to be regular if it is generated through a set of defining relations. In this article all the designs are assumed to belong to the class regular FFDs. In an FFD the numbers *1, 2, ……, n* assigned to the factor are referred to as letters, while the product formed by any subset of these letters is termed a word. If A_i denotes the number of words of length *i* appearing in the defining relation of a design *d*, then the number of letters contained in

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a word is called word length and the sequence $A(d) = (A_1, A_2, \ldots\ldots\ldots, A_k)$ is known as the word length pattern of design.

Definition 2.1. Box and Hunter (1961) defined "the resolution of a design d is as the minimum value r such that $A_r \geq 1$. In practical terms, a design is said to be of resolution r if no interaction involving p factors is confounded with interactions involving fewer than ($r-p$) factors."

Definition 2.2. Box and Hunter (1961) defined that "Alias is a condition where two or more effects or treatment contrasts (main effects or interaction) become completely indistinguishable from each other". This happens in FFD because running only a fraction of full experiment limits the unique data available forestimating the effects independently, causing different effects to share the same statistical formulae.

Example 2.1. For the 2^{6-2}design with generators $5 = 12$ and $6 = 34$, the complete defining relation is given as:

$$I = 125 = 346 = 123456$$

The expressions125, 346, and 123456 the generators or *words* of the design, *Word length* is 3, 3 and 6 for each word respectively (it is the total number of letter/digits appearing in the generator).

Since the shortest words in the above defining relation contain three letters, the design is classified as a *Resolution III* design and word length pattern is *A(d)* = (2,0,0,1). Alias structure for 2^{6-2} design with generators 5 = 12 and 6 = 34 is given in Table 1.

Table 1: Alias Structure for 2^{6-2} design

Alias Table			
I	**125**	**346**	**123456**
1	25	1346	23456
2	15	2346	13456
3	1235	46	12456
4	1245	36	12356
5	12	3456	12346

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6	1256	34	12345
13	235	146	2456
14	245	136	2356
16	256	134	2345
23	135	246	1456
24	145	236	1356
26	156	234	1345
35	123	456	1246
45	124	356	1236
56	126	345	1234

Designs with equal resolution may not always be equally good. Thus, Fries and Hunter (1980) introduced the *MA* criterion to select between designs with equal resolution.

Definition 2.3. According to Cheng et al. (1992) for two designs d_1 and d_2, let s be the smallest integer for which $A_s(d_1) \neq A_s(d_2)$. Then design d_1 is said to have less aberration than design d_2, denoted by $A(d_1) < A(d_2)$, if there is no other design with less aberration than d_1, then d_1 is referred to as a *MA* design.

Example 2.2: Consider 2^{6-2} designs d_1 and d_2 with defining relations I = 125 = 1346 = 23456 and I = 125 = 346 = 123456 respectively. The word length pattern for design d_1 and d_2 are

$$A(d_1) = (1,1,1,0)$$

$$A(d_2) = (2,0,0,1)$$

Both the designs are resolution III designs but design d_1 is *MA* design ($A(d_1) < A(d_2)$), as design d_1 has three *m.e.'s* aliased with 2 *fi's* (1 and 25, 2 and 15, 5 and 12) while design d_2 has six *m.e.'s* aliased with 2 *fi's* (1 and 25, 2 and 15, 3 and 46, 4 and 36, 5 and 12, and 6 and 34).

The *MA* criterion is commonly used as a criterion for selecting optimal 2^{n-p} FFD. Nevertheless, when designs of resolution V or higher are not feasible this criterion alone may lead to best design. Chen et al. (1993) suggested that maximum resolution and *MA* criteria do not completely describe the existence of clear 2 *fi's* in designs with resolutions III and IV.

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Therefore, according to the estimation capacity, the design with the maximum clear *m.e.'s* and greatest possible number of 2 *fi's* are preferred.

Definition 2.4. A main effect or 2 *fi* is said to be clear if it is not aliased with any other main effect and 2 *fi's* and can be estimated independently of other main effects and other 2 *fi's.*

Foldover designs, introduced by Box et al. (1978), are a classical follow-up technique in experimental design used to de-alias confounded effects.

Definition 2.5. A design is a foldover design (Box et al. 1978) when the fraction obtained by reversing the signs of one or more columns of the initial design is added to the initial design.

Li and Lin (2003) defined a foldover plan as a set of factors whose signs are to be reversed in a foldover design. To illustrate this, consider the design 2^{6-2} (with generators 5 = 12 and 6 = 34) and select a foldover plan in which the signs of factors 5 and 6 are reversed. According to this all the words in the defining relation that contain 5 but do not contain 6 or contain 6 but do not contain 5 will have their signs reversed. However, if a word contains both 5 and 6, its sign remains unchanged, since two negatives when multiplied will give positive sign and product of positives remains same. As a result, the defining relation of the new fraction becomes I = -125= -346 = 123456.

For a resolution III design, when the signs of all the factors are reversed it will result in a combined foldover design which has resolution IV.

Definition 2.5.1. Li and Lin (2003) stated that "core foldover plans are foldover plans containing only the generators and their possible combinations".

For 2^{5-2} design the core foldover plan is (0, 4, 5, 45), where 0 means no factor sign is reversed, 4 means the sign of factor 4 is reversed and so on. A generalisation is given in the following theorem by Li and Lin (2003):

Theorem 2.1. Li and Lin (2003) stated that "for a 2^{n-p} design with p generators $G_1, G_2, \ldots, Gp$, any foldover plan is equivalent to a core foldover plan. Moreover, for every core foldover plan, there are 2^{n-p} foldover plans that are equivalent to it"**.**

This theorem shows that for each core foldover plan there are 2^{n-p} foldover plans associated with it, that are equivalent to it. Therefore, the equivalent foldover plans defined by Li and Lin

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results in designs with same defining relation and hence same *MA*. This theorem implies that for every core foldover plan, there are 2^{n-p} foldover plans that are equivalent to it.

When evaluating the cost of an experiment, the number of level changes (NLC) in a design is essential since some level changes can be quite expensive, particularly in industrial experiments, and some level changes may not even be necessary for the design. Therefore, designs that have fewer level changes could be more economical.

Definition 2.6. The design in which the number of level changes is less as compared to all other designs with the same parameters is known as the minimum level change design.

In some situations, experimental schemes may suffer from unknown and uncontrollable sources of variation that may have high correlation with the time or position in which the observations are taken. When such trends are present in a fractional factorial experiment, they can affect the estimation of main effects as well as interaction effects, thereby influencing the accuracy of the experimental results. In such cases, the run order of the observations may be chosen so that estimates of main effects and interactions are trend robust. A trend-free design is a design in which the ordering of the treatments is such that the trend does not influence the effects.

Definition 2.7. Let $y = (y_1, y_2, \dots \dots . y_N)'$ represent the ordered vector of observations, $T_x = (1^x, 2^x, \dots \dots N^x)'$for $x = 0,1,2, \dots \dots v$, T_x be the N × 1 vector of trend coefficients of order *x*. Further, let u_i denote the contrast vector corresponding to for main effect A_i, where i = 1, 2, ... *n*, arranged according to the run order. Then the quantity $u_i' T_x$ is known as the time count associated with the *m.e.* A_i .

A *m.e.* contrast u_i is said to be v-trend free if and only if it satisfies

$$u_i' T_x = 0 \qquad \forall x = 0,1, \dots \dots \dots , v \tag{1}$$

In general, an N × 1 vector u is called v-trend free if condition (1) is satisfied.

3. Optimal foldover plans based on *MA* and clear factor interaction

Li and Lin (2003) investigated optimal foldover plans under the minimum aberration (*MA*) criterion. They introduced the concept of core foldover plans (Definition 2.5.1) and discussed about equivalent foldover designs (Theorem 2.1).

In the present work, the optimal foldover designs proposed by Li and Lin (2003) are further examined using the criterion of clear factor interactions. A comparative analysis of these designs with respect to the above mention criteria is presented in Tables 2, 3, and 4.
Table 2 show that the foldover plans (5, 6, 56) of design 6-2.1 have *MA* and for these designs all main effects and nine clear 2 *fi's* are estimable. Whereas in design 6-2.2, only foldover plan 56 has *MA* and for this design, all main effects and 2 *fi's* are clearly estimable. In design 6-2.3, the foldover plan 56 is the only plan with *MA* and has all main effects and 2 *fi's* are clearly estimable. If all three designs of 2^{6-2} are compared, the foldover plan 56 of 6-2.3 design has the *MA* and for this design, all main effects and 2 *fi's* are clearly estimable. Thus, the foldover plan 56 of the design 6-2.3 is optimal.

Table 2: $2^{6-2}.i$ ($1 \le i \le 3$) FFD with respective generators and clear factor interactions

k-p	generator	Core Foldover plan	Word length pattern	Clear factor interaction	
				Main effects	2 *fi's*
6-2.1	5=123, 6=124	5	(0,1,0,0)	6	9
		6	(0,1,0,0)	6	9
		56	(0,1,0,0)	6	9
6-2.2	5=12, 6=134	5	(0,1,0,0)	6	9
		6	(1,0,0,0)	3	12
		56	(0,0,1,0)	6	15
6-2.3	5=12, 6=34	5	(1,0,0,0)	3	12
		6	(1,0,0,0)	3	12
		56*	(0,0,0,1)	6	15

Table 3, shows that foldover plans (5, 6, 7, 56, 57, 67, 567) have *MA* for design 7-3.1 and all the *m.e.'s* and six 2 *fi's* are estimable. In design 7-3.2 and 7-3.3, only the foldover plan 567 has *MA* and all the main effects clearly estimable. For design 7-3.2, foldover plan 567, fifteen 2 *fi's* are clearly estimable and for design 7-3.3, foldover plan 567, nine 2 *fi's* are clearly estimable. Similarly, in design 7-3.4 and 7-3.5, foldover plan 567 has *MA* and all the *m.e.'s* and

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six 2 *fi's* are clearly estimable. The design 7-3.2 stands out because it not only has *MA* and all *m.e.'s* estimable, but it also has fifteen 2 factor interactions clearly estimable which is highest among the compared designs. Therefore, based on the criteria of *MA* and estimability of effects (main effects and two factor interactions), design 7-3.2 with the foldover plan 567 is considered the optimal choice among the designs listed in Table 3. This conclusion is drawn from the fact that it offers the best balance of these criteria compared to the other designs evaluated.

In Table 4, the foldover plans (56, 57, 58, 67, 68, 78, 5678) for 8-4.1 design have MA, it can clearly estimate all the *m.e.'s* with no 2*fi's*, while the foldover plans (5, 6, 7, 8, 567, 568, 578, 678) are not *MA* designs but can clearly estimate all the *m.e.'s* with seven clear *2fi's*. For design 8-4.2 and 8-4.3, the foldover plan 5678 has *MA* and can clearly estimate all main effects. Design 8-4.2, the foldover plan 5678 can estimate seven clear 2*fi's* and design 8-4.3, the foldover plan 5678 four clear 2*fi's*. In design 8-4.4 and 8-4.6, the foldover plan 567 has *MA* and can clearly estimate all *m.e.'s*. Foldover plan 567 of 8-4.4 can estimate thirteen clear 2*fi's* and foldover plan 567 of 8-4.6 can estimate seven clear 2*fi's*.

Table 3: Table for $2^{7-3}.i$ ($1 \le i \le 5$) FFD with respective generators and clear factor interaction

k-p	**generator**	**Core Foldover plan**	**Word length pattern**	**Clear factor interaction**	
				Main effects	**2 *fi's***
7-3.1	5=123,6=124,7=134	5	(0,3,0,0,0)	7	6
		6	(0,3,0,0,0)	7	6
		7	(0,3,0,0,0)	7	6
		56	(0,3,0,0,0)	7	6
		57	(0,3,0,0,0)	7	6
		67	(0,3,0,0,0)	7	6
		567	(0,3,0,0,0)	7	6
7-3.2	5=12,6=13,7=234	5	(1,1,1,0,0)	4	12
		6	(1,1,1,0,0)	4	12
		7	(2,1,0,0,0)	2	11
		56	(0,3,0,0,0)	7	6
		57	(1,1,1,0,0)	4	12
		67	(1,1,1,0,0)	4	12
		567*	(0,1,2,0,0)	7	15

7-3.3	5=12,6=13,7=24	5	(2,0,0,1,0)	1	15
		6	(2,1,0,0,0)	2	11
		7	(2,1,0,0,0)	2	10
		56	(1,1,1,0,0)	4	12
		57	(1,1,1,0,0)	4	12
		67	(1,0,1,1,0)	4	18
		567	(0,2,0,1,0)	7	9
7-3.4	5=12,6=13,7=14	5	(2,1,0,0,0)	2	11
		6	(2,1,0,0,0)	2	10
		7	(2,1,0,0,0)	2	11
		56	(1,1,0,0,1)	4	12
		57	(1,1,0,0,1)	4	12
		67	(1,1,0,0,1)	4	12
		567	(0,3,0,0,0)	7	6
7-3.5	5=12,6=13,7=23	5	(2,1,0,0,0)	2	11
		6	(2,1,0,0,0)	2	11
		7	(2,1,0,0,0)	2	11
		56	(2,1,0,0,0)	2	11
		57	(2,1,0,0,0)	2	11
		67	(2,1,0,0,0)	2	11
		567	(0,3,0,0,0)	7	6

Table 4: $2^{8-4}.i$ ($1 \leq i \leq 6$) FFD with respective generators and clear factor interaction

k-p	**generator**	**Core Foldover plan**	**Word length pattern**	**Clear factor interaction**	
				Main effects	**2 *fi's***
8-4.1	5=123,6=124,7=134,8=234	5	(0,7,0,0,0,0)	8	7
		6	(0,7,0,0,0,0)	8	7
		7	(0,7,0,0,0,0)	8	7
		8	(0,7,0,0,0,0)	8	7
		56	(0,6,0,0,0,1)	8	0
		57	(0,6,0,0,0,1)	8	0
		58	(0,6,0,0,0,1)	8	0
		67	(0,6,0,0,0,1)	8	0
		68	(0,6,0,0,0,1)	8	0
		78	(0,6,0,0,0,1)	8	0
		567	(0,7,0,0,0,0)	8	7

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		568	(0,7,0,0,0,0)	8	7
		578	(0,7,0,0,0,0)	8	7
		678	(0,7,0,0,0,0)	8	7
		5678	(0,6,0,0,0,1)	8	0
8-4.2	5=12,6=13,7=14,8=234	5	(2,3,2,0,0,0)	3	9
		6	(2,3,2,0,0,0)	3	9
		7	(2,3,2,0,0,0)	3	9
		8	(3,3,0,0,1,0)	1	7
		56	(1,3,2,0,1,0)	5	10
		57	(1,3,2,0,0,1)	5	10
		58	(2,3,2,0,0,0)	3	9
		67	(1,3,2,0,1,0)	5	10
		68	(2,3,2,0,0,0)	3	9
		78	(2,3,2,0,0,0)	3	9
		567	(0,7,0,0,0,0)	8	7
		568	(1,3,2,0,1,0)	5	10
		578	(1,3,2,0,1,0)	5	11
		678	(1,3,2,0,1,0)	5	10
		5678*	(0,3,4,0,0,0)	8	13
8-4.3	5=12,6=13,7=24,8=34	5	(3,2,1,1,0,0)	1	11
		6	(3,2,1,1,0,0)	1	11
		7	(3,2,1,1,0,0)	1	11
		8	(3,2,1,1,0,0)	1	11
		56	(2,3,2,0,0,0)	3	9
		57	(2,3,2,0,0,0)	3	9
		58	(2,1,2,2,0,0)	2	16
		67	(2,1,2,2,0,0)	2	16
		68	(2,3,2,0,0,0)	3	9
		78	(2,3,2,0,0,0)	3	9
		567	(1,2,3,1,0,0)	5	13
		568	(1,2,3,1,0,0)	5	13
		578	(1,2,3,1,0,0)	5	13
		678	(1,2,3,1,0,0)	5	9
		5678	(0,5,0,2,0,0)	8	4
8-4.4	5=12,6=13,7=23,8=1234	5	(2,3,2,0,0,0)	3	9
		6	(2,3,2,0,0,0)	3	9
		7	(2,3,2,0,0,0)	3	9
		8	(4,3,0,0,0,0)	2	13
		56	(2,2,2,0,0,1)	3	12

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		57	(2,2,2,0,0,1)	3	12
		58	(2,2,2,0,0,1)	3	12
		67	(2,2,2,0,0,1)	3	12
		68	(2,2,2,0,0,1)	3	12
		78	(2,2,2,0,0,1)	3	12
		567*	(0,3,4,0,0,0)	8	13
		568	(2,3,2,0,0,0)	3	11
		578	(2,3,2,0,0,0)	3	9
		678	(2,3,2,0,0,0)	3	9
		5678	(0,6,0,0,0,1)	8	0
8-4.5	5=12,6=13,7=23,8=14	5	(3,2,1,1,0,0)	1	11
		6	(3,2,1,1,0,0)	1	11
		7	(3,3,0,0,1,0)	1	7
		8	(4,3,0,0,0,0)	2	8
		56	(3,1,0,2,1,0)	0	15
		57	(3,2,1,1,0,0)	1	11
		58	(2,2,1,1,1,0)	3	12
		67	(3,2,1,1,0,0)	1	11
		68	(2,2,1,1,1,0)	3	12
		78	(2,1,2,2,0,0)	3	18
		567	(1,3,2,0,1,0)	5	10
		568	(2,3,2,0,0,0)	3	9
		578	(2,2,1,1,1,0)	3	12
		678	(2,2,1,1,1,0)	3	11
		5678	(0,5,0,2,0,0)	8	4
8-4.6	5=12,6=13,7=23,8=123	5	(4,3,0,0,0,0)	2	7
		6	(4,3,0,0,0,0)	2	7
		7	(4,3,0,0,0,0)	2	7
		8	(4,3,0,0,0,0)	2	7
		56	(3,3,0,0,1,0)	1	7
		57	(3,3,0,0,1,0)	1	7
		58	(3,3,0,0,1,0)	1	7
		67	(3,3,0,0,1,0)	1	7
		68	(3,3,0,0,1,0)	1	7
		78	(3,3,0,0,1,0)	1	7
		567	(0,7,0,0,0,0)	8	7
		568	(4,3,0,0,0,0)	2	13
		578	(4,3,0,0,0,0)	2	13
		678	(4,3,0,0,0,0)	2	13
		5678	(3,3,0,0,1,0)	1	7

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* indicates the design which is best in according to both MA and maximum clear *m.e.'s* and 2 *fi's*.

In design 8-4.5, the foldover plan 5678 has *MA* and can clearly estimate all *m.e.'s* and four clear 2 *fi's*. Thus, comparing all the designs of 2^{8-4}, the fold over plan 5678 of design 8-4.2 and 567 of design 8-4.4 are optimal according to the *MA* criterion and the maximum number of clear 2 *fi's.*

4. Optimal trend free foldover plans with minimum number of level changes

According to Li and Lin (2003), for every core foldover plan, there are 2^{n-p} foldover plans that are equivalent to the core foldover plan, hence it is sufficient to use the representative core foldover plan in each group. In this section, we classify the equivalent designs based on the number of changes in the factor levels and the number of linear trend-free factors. The equivalent designs for 2^{6-2} were generated using Table 5 with generator 5=123, 6=124, (for 2^{7-3}, 2^{8-4} the equivalent designs are available with the authors) and the NLC and LTF factors for *m.e.'s* and 2 *fi's* for each design were calculated. Since there are very large number of equivalent designs (from Table 5, corresponding to each foldover plan there are 16 plans that are equivalent to it), so only those plans were considered which have all the *m.e.'s* and 2 *fi's* trend-free and simultaneously have least number of changes in factor levels.

Table 5: Equivalent design for 2^{6-2}FFD with generator 5=123, 6=124

	0	1	2	3	4	12	13	14	23	24	34	123	124	134	234	1234
0	0	156	256	35	46	12	136	145	236	245	3456	1235	1246	134	234	123456
5	5	16	26	3	456	125	1365	14	1356	24	346	123	12456	1345	2345	12346
6	6	15	25	356	4	126	13	1456	23	2456	345	12356	124	1346	2346	12345
56	56	1	2	36	45	1256	135	146	235	246	34	1236	1245	13456	23456	1234

In Table 5 each row shows a foldover plan which is equivalent to core foldover plan.

These equivalent core foldover plans are further classified based on the NLC and the number of linear trend-free factors (main and 2 *fi's*). In Table 5, foldover plans *5* and *12346* are equivalent (because both have the same defining relation) but, for foldover plan *5* the total NLC is 342, two main effects and fifteen 2 *fi's* are linear trend-free, whereas foldover plan *12346*

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has 338 level changes and for this all main effects and all 2 *fi's* are trend-free. Similarly, in other cases (i.e. for *0, 6, 56*), each foldover plan equivalent to core foldover plan have different NLC and the number of LTF factors. These foldover plans which are equivalent based on defining relation are not equivalent according to the NLC and the number of linear trend-free factors. Table 6, 7 and 8, present equivalent foldover plans which have the minimum NLC and have all main effects and 2 *fi's* linear trend-free.

Table 6: 2^{6-2} design with their respective NLC and LTF

6-2.1		
Optimal foldover design	**No. of level changes**	**Linear trend free factors**
123456	338	All *m.e's* and 2 *fi's*
12346	338	
12345	338	
1234	342	
6-2.2		
12346	359	All *m.e's* and 2 *fi's*
1234	362	
6-2.3		
1234*	**312**	All *m.e's* and 2 *fi's*

NLC and LTF are number of level changes and linear trend free factor respectively.
* Shows the best design according to the minimum number of level changes and trend free criteria.

In Table 6, design 6-2.1, the foldover plans 123456, 12346, 12345 and 1234 are equivalent to core foldover plan (0,5,6,56) respectively but have different numbers of level changes. These plans have all the main effects and 2 *fi's* linear trend-free while other equivalent designs corresponding to the core foldover plan have a lesser number of linear trend-free factors. Similarly, in designs 6-2.2 and 6-2.3, Table 6 shows the designs, which have the minimum NLC. If we compare all the designs of 2^{6-2}, then from Table 6, it can be seen that the foldover plan 1234 of design 6-2.3 has the minimum NLC.

Table 7: 2^{7-3} design with their respective NLC and LTF

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7-3.1		
Optimal foldover design	**No. of level changes**	**Linear trend free factors**
1234567	461	All *m.e's* and 2 *fi's*
123467	461	
123457	461	
123456	461	
12347	466	
12346	466	
12345	466	
1234	469	
7-3.2		
12347*	458	All *m.e's* and 2 *fi's*
1234	465	
7-3.3		
12347	464	All *m.e's* and 2 *fi's*
7-3.4		
1234	480	All *m.e's* and 2 *fi's*
7-3.5		
1234	460	All *m.e's* and 2 *fi's*

Table 7, mentions those designs that have the minimum NLC and all the main effects and 2 *fi's* linear trend-free. If we compare all the designs of 2^{7-3}, then it can be seen from Table 7 that the foldover plan 12347 of design 7-3.2 has minimum NLC.

Table 8: 2^{8-4} design with their respective NLC and LTF

8-4.1		
Optimal foldover design	**No. of level changes**	**Linear trend free factors**
12345678	576	All *m.e's* and 2 *fi's*
1234678	584	
1234578	584	
1234568	584	
1234567	584	
123478	590	

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123468	590	
123467	590	
123458	590	
123457	590	
123456	590	
12348	594	
12347	594	
12346	594	
12345	594	
1234	596	
	8-4.2	
Optimal foldover design	**No. of level changes**	**Linear trend free**
12348	600	All *m.e's* and 2 *fi's*
	8-4.3	
1234*	568	All *m.e's* and 2 *fi's*
	8-4.4	
1234	600	All *m.e's* and 2 *fi's*
12348	608	
	8-4.5	
1234	600	All *m.e's* and 2 *fi's*
	8-4.6	
12348	600	All *m.e's* and 2 *fi's*

Table 8, mentions all design which have minimum NLC and all the main effects and 2*fi's* LTF. If we compare all the designs of 2^{8-4}, the foldover plan 1234 of design 8-4.3 has minimum NLC.

Conclusion

It can be observed from the present study that, Li and Lin (2003), proposed equivalent foldover plans that result in designs that are equivalent with respect to the minimum aberration criterion. However, this equivalence does not necessarily extend to other optimality criteria, such as clear factor interactions (*m.e's* and 2*fi's*), the NLC and trend-free properties of factors. Therefore, designs that are comparable under the minimum aberration framework may still differ substantially when evaluated using other optimality criteria discussed in this paper.

The analysis identifies four foldover plans — plan 56 of design 6-2.3, plan 567 of design 7-3.2, plan 5678 of design 8-4.2, and plan 567 of design 8-4.4 — as superior designs, excelling not only under the minimum aberration criterion but also in terms of clear main effects and two-factor interaction estimation. Separately, three equivalent foldover plans - plan 1234 of design 6-2.3, plan 12347 of design 7-3.2, and plan 1234 of design 8-4.3 - emerge as optimal with respect to minimizing the NLC while simultaneously maximizing the number of linear trend-free factors which include all clear main effects and two-factor interactions.

The study further reveals that no single foldover plan simultaneously satisfies all four criteria, namely minimum aberration, clear factor interaction estimation, minimum NLC, and maximum linear trend-free factors. Hence, the choice of an appropriate design depends on the specific experimental objectives and practical considerations. Different experimental scenarios will therefore call for different design choices, as no universally optimal foldover plan exists across all evaluation criteria.

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